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ABSTRACT

The problem is concerned with the hydrodynamics of a submerged jet taking into account the initial discharge and generalized for the case of a nonisothermal jet.

The problem concerning the isothermal jet with an assigned impulse was solved in (ref. 1) where the initial discharge was assumed to be equal to zero, while the impulse was considered to finite. The effect of a finite initial discharge was investigated for an untwisted (ref. 2) and twisted (ref. 3) jet. The nonisothermal problem of a round jet with a given impulse and a given quantity of heat but without taking into account the finite value of the initial discharge was solved by Chia Shun Yih (ref. 4). The present article presents the solution of the problem on the propagation of the submerged nonisothermal round jet with a given discharge, impulse and quantity of heat. /106*

*Numbers given in the margin indicate the pagination in the original foreign text.

The equations which describe this process in a cylindrical system of coordinates, with the origin at the center of the output section, have the form (ref. 4)

$$
\begin{gathered}
u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right),\\
\frac{\partial (ru)}{\partial x}+\frac{\partial (rv)}{\partial r}=0,\\
u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial r}=\frac{\nu}{\sigma}\left(\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}\right),\\
\frac{\partial u}{\partial r}=0,\quad v=0,\quad \frac{\partial \theta}{\partial r}=0 \text{ and } r=0,\\
u\to 0,\quad \theta\to 0 \text{ and } r\to\infty.
\end{gathered}
\tag{1}
$$

For a nontrivial solution of system (1) it is necessary to satisfy additional integral conditions for the conservation of impulse and quantity of heat:

$$2\pi\rho\int_0^\infty ru^2\,dr=I_0=\text{const}, \tag{2}$$

$$2\pi T_\infty\int_0^\infty ru\theta\,dr=H_0=\text{const}. \tag{3}$$

For the hydro-dynamic part of the problem which may be considered separately if we assume that the temperature drop is small, we obtained solutions in the form of asymptotic expansions using the inverse powers of the abscissas (ref. 5): /107

$$
\begin{gathered}
u=\frac{3}{8\pi}\frac{I_0}{\nu\rho}\frac{1}{\left(1+\frac{1}{4}\eta^2\right)^2}\frac{1}{x}-\frac{3}{64\pi^2}\frac{M_0 I_0}{\nu^2\rho^2}\frac{1-\frac{3}{4}\eta^2}{\left(1+\frac{1}{4}\eta^2\right)^3}\frac{1}{x^2}+\ldots,\\
v=\sqrt{\frac{3}{16}\frac{I_0}{\rho}}\,\frac{\eta\left(1-\frac{1}{4}\eta^2\right)}{\left(1+\frac{1}{4}\eta^2\right)^2}\frac{1}{x}-\\
-\frac{M_0}{16}\sqrt{\frac{3I_0}{\pi^2\rho^2\nu^2}}\cdot\frac{\eta\left(1-\frac{3}{4}\eta^2\right)}{\left(1+\frac{1}{4}\eta^2\right)^3}\frac{1}{x^2}+\ldots,
\end{gathered}
\tag{4}
$$

where

$$\eta=\left(\frac{3I_0}{16\pi\rho\nu^2}\right)^{1/2}\frac{r}{x}. \tag{5}$$

We shall seek the solution of the third equation of system (1) in the form of a similar asymptotic expansion

$$\theta = \frac{H_0}{\nu T_\infty}\,\frac{1}{x}\,\theta_0(\eta) + \frac{H_0^2}{\nu^2 T_\infty^2}\,\frac{1}{x^2}\,\theta_1(\eta) + \ldots \qquad (6)$$

Having determined the derivatives $\partial\theta/\partial x$, $\partial\theta/\partial r$, $\partial^2\theta/\partial\eta^2$ from (6) and substituted them together with expressions for u and v (4) into the third equation of system (1) we equate the coefficients in front of the same negative powers of x in both parts; then we obtain the following system of ordinary differential equations of the second order (the prime indicates that the derivative is taken with respect to η):

$$\eta\theta_0'' + \theta_0' + \sigma\left[\frac{2\eta\theta_0}{\left(1+\frac{1}{4}\eta^2\right)^2} + \frac{\eta^2\theta_0'}{1+\frac{1}{4}\eta^2}\right] = 0,$$

$$\eta\theta_1'' + \theta_1' + \sigma\left[\frac{4\eta\theta_1}{\left(1+\frac{1}{4}\eta^2\right)^2} + \frac{\eta^2\theta_1'}{1+\frac{1}{4}\eta^2}\right] =$$

$$= \frac{\sigma}{4\pi}\,\frac{M_0 T_\infty}{\rho H_0}\,\frac{\eta\left(1-\frac{3}{4}\eta^2\right)}{\left(1+\frac{1}{4}\eta^2\right)^3}\,\theta_0. \qquad (7)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\theta_0' = \theta_1' = \ldots = 0 \text{ and } \eta = 0,$$

$$\theta_0 = \theta_1 = \ldots = 0 \text{ and } \eta \to \infty.$$

A comparison of coefficients in front of the same powers of x under /108
conditions of nontriviality (3) leads to the following integral relationships:

$$\int_0^\infty \frac{\eta\theta_0}{\left(1+\frac{1}{4}\eta^2\right)^2}\,d\eta = \frac{1}{4\pi};$$

$$\int_0^\infty \left[\frac{\eta\theta_1}{\left(1+\frac{1}{4}\eta^2\right)^3} - \frac{1}{8\pi}\,\frac{M_0 T_\infty}{\rho H_0}\,\frac{\eta\left(1-\frac{3}{4}\eta^2\right)}{\left(1+\frac{1}{4}\eta^2\right)^3}\,\theta_0\right] d\eta = 0. \qquad (8)$$

The first equation of system (7) was solved by Chia Shun Yih (ref. 4). He found that

$$\theta_0 = \frac{2\nu+1}{8\pi}\left(1+\frac{1}{4}\eta^2\right)^{-2\nu}. \tag{9}$$

To find θ, we transform to a new argument ξ in accordance with the equation

$$\xi = \frac{1}{4}\eta^2\left(1+\frac{1}{4}\eta^2\right)^{-1}. \tag{10}$$

Then we shall have (the dot above the function means that the derivative is taken with respect to ξ)

$$\xi(1-\xi)\ddot{\theta}_1 + [1+2(\nu-1)\xi]\dot{\theta}_1 + 4\nu\theta_1 = \beta(1-\xi)^{2\nu}(1-4\xi), \tag{11}$$

where

$$\beta = \frac{\sigma(2\nu+1)}{32\pi^2}\,\frac{M_0 T_\infty}{\rho H_0}. \tag{12}$$

The second condition of system (8) will take the form

$$\int_0^1 \theta_1(\xi)\,d\xi = \beta\frac{\sigma-1}{2\sigma(\sigma+1)(2\nu+1)}. \tag{13}$$

It is easy to see that the solution of the homogeneous equation (11) satisfying the condition of finiteness for $\xi = 0$ will be the power series

$$\bar{\theta}_1 = \sum_{k=0}^{\infty} A_k \xi^k, \tag{14}$$

where $A_0 = 1$ while the other coefficients are determined by the recurrent relationship

$$A_{k+1} = \frac{k(k+1) - 2\nu(k+2)}{(k+1)^2}A_k. \tag{15}$$

Series (14) converges when $\xi < 1$, i.e., for all finite values of η. The particular solution of the nonhomogeneous equation (11) will also be sought in the form of a power series

$$\bar{\bar{\theta}}_1 = \beta\sum_{k=0}^{\infty} B_k \xi^k. \qquad (16)$$

The coefficients of this series are determined by the following recurrent relationship /109

$$B_{k+1} = -\frac{[k(k+1) - 2\sigma(k+2)]B_k + (a_k - 4a_{k-1})}{(k+1)^2}, \qquad (17)$$

where a_k is the coefficient in front of ξ^k for the binomial expansion $(1 - \xi)^{2a}$ in a power series, while $B_0 = 0.5$, $a_{-1} = 0$. Series (17) also converges when $\xi < 1$.

Taking into account (14) and (16) we obtain the general solution of differential equation (11) in the form

$$\theta_1 = C\sum_{k=0}^{\infty} A_k \xi^k + \beta\sum_{k=0}^{\infty} B_k \xi^k. \qquad (18)$$

Substituting expansion (18) into the integral condition (13), we obtain an expression for the constant of integration C:

$$C = \frac{\beta}{\sum\limits_{k=0}^{\infty}\frac{A_k}{k+1}}\left[\frac{\sigma-1}{2\sigma(\sigma+1)(2\sigma+1)} - \sum_{k=0}^{\infty}\frac{B_k}{k+1}\right]. \qquad (19)$$

After this we can write the final expression for $\theta(x,\eta)$,

$$\begin{aligned}\theta(x,\eta) = {} & \frac{2\sigma+1}{8\pi}\,\frac{H_0}{\nu T_w}\left(1+\frac{1}{4}\eta^2\right)^{-2\sigma}\frac{1}{x} + \\ & + \frac{\sigma(2\sigma+1)}{32\pi^2}\,\frac{H_0}{\nu T_w}\,\frac{M_0}{\rho}\left\{\left(\sum_{k=0}^{\infty}\frac{A_k}{k+1}\right)^{-1}\times\right. \\ & \times\left[\frac{\sigma-1}{2\sigma(\sigma+1)(2\sigma+1)} - \sum_{k=0}^{\infty}\frac{B_k}{k+1}\right]\sum_{k=0}^{\infty}A_k\left[\frac{1}{4}\eta^2\left(1+\frac{1}{4}\eta^2\right)^{-1}\right]^k + \\ & \left. + \sum_{k=0}^{\infty}B_k\left[\frac{1}{4}\eta^2\left(1+\frac{1}{4}\eta^2\right)^{-1}\right]^k\right\}\frac{1}{x^2} + \dots \end{aligned} \qquad (20)$$

In a particular case when $\sigma = 1$ the homogeneous part of equation (11) has the form

$$\xi(1-\xi)\ddot{\theta}_1 + \dot{\theta}_1 + 4\theta_1 = 0. \qquad (21)$$

Since the coefficient in front of the unknown function in equation (21) is not one of the natural numbers n(n-1), where n = 0, 1, 2..., the only limited solution for $\sigma = 1$ will be the zero solution (6). Therefore, the general solution of the equation in this case will consist only of the particular solution which in the case considered is transformed into a polynomial of the third degree

$$\theta_1 = \beta[-0.5 + 3\xi - 4.5\xi^2 + 2\xi^3] \qquad (22)$$

Transforming to the initial argument η we obtain for $\sigma = 1$

$$\theta_1 = -\frac{\beta}{2}\left(1 - \frac{3}{4}\eta^2\right)\left(1 + \frac{1}{4}\eta^2\right)^{-3}. \qquad (23)$$

while θ will be given by the expansion /110

$$\theta = \frac{3}{8\pi}\,\frac{H_0}{\nu T_\infty}\,\frac{1}{\left(1 + \frac{1}{4}\eta^2\right)^2}\,\frac{1}{x} -$$
$$- \frac{3}{64\pi^2}\,\frac{H_0}{\nu T_\infty}\,\frac{M_0}{\mu}\left(1 - \frac{3}{4}\eta^2\right)\left(1 + \frac{1}{4}\eta^2\right)^{-3}\frac{1}{x^2} + \cdots \qquad (24)$$

A comparison of equalities (4) and (24) shows that in this case as well as in the case when the initial discharge is not taken into account, there is a similarity between the distribution of longitudinal velocities and temperatures.

Equation (20) was used to carry out calculations for three values of the initial discharge (figs. 1-3) with $\sigma = 0.72$.

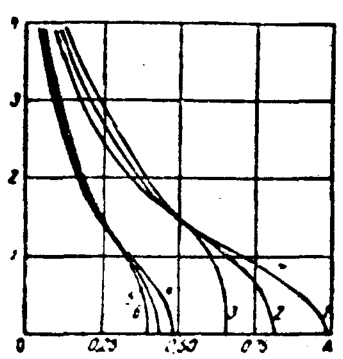

Figure 1. The variation in the dimensionless temperature at two cross sections of the jet for $x = 0.1$ ($1 - M, \mu = 0$, $2 - 0.4$, $3 - 0.8$); 0.2 ($4 - M, \mu = 0$, $5 - 0.4$, $6 - 0.8$) $A = 1.021 \frac{\sqrt{T_\infty}}{H_0} \theta(\eta)$

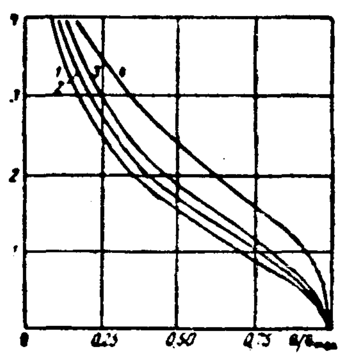

Figure 2. The ratio $\theta(\tau)/\theta_{max}$ and its maximum value θ_{max} 1 for M $\mu = 0$; 2 and 3 for $x = 0.2$ and M, μ = 0.4 and 0.8; 3 and 4 for $x = 0.1$ and M μ = 0.4 and 0.8.

From the curves which are presented we can conclude that as the initial discharge increases the curves $\theta(\eta)$ $\theta(\eta)/\theta_{max}$ become more filled and the effect of the initial discharge decreases when the distance from the outlet increases.

The solution which has been obtained shows the possibility of applying the asymptotic expansion in the case of a nonisothermal jet.

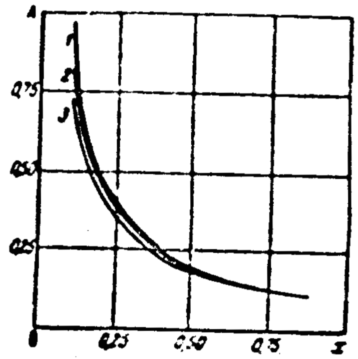

Figure 3. Variation in θ_{max} along the axis of the jet 1 for M_e $\mu = 0$, 2 - 0.1, 3 - 0.3, 4 - 1.001 $\frac{T_\infty}{M_e}$

Symbols Used

x, rnu, v are the longitudinal and radial coordinates and velocities respectively; T - is the temperature at any point in the jet; T_∞ - is the temperature of the liquid in which the jet is propagated, $(T - T_\alpha)/T_\infty$ is the dimensionless temperature; ρ is the fluid density, σ is the Prandtl number, η, μ are the kinematic and dynamic coefficients of viscosity, M_0 is the initial discharge.

Summary

The paper considers the problem of a submerged nonisothermal round jet taking into account the initial mass-flow rate. The method of asymptotic expansion is used to obtain the solution. The solution is used to carry out calculations for three values of the initial discharge into two sections of the jet with $\sigma = 0.72$. The relationship $\theta(\eta)$ is shown in figure 1, the ratio $\theta(\tau)/\theta_{max}$ is shown in figure 2 while the variation in θ_{max} along the axis is shown in figure 3.

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